WORLD FORUM ON ACADEMIC INNOVATION AND DIGITAL LITERACY

CALCULATION OF THE NORMAL DEPTH IN AN IRRIGATION CANAL USING NEWTON'S METHOD

UDC 519.61:532.543

Khamroeva Dilafruz Namozovna,
PhD in Physical and Mathematical Sciences, Associate Professor,
Department of Digital Technologies, Navoi State University
ORCID ID: 0009-0000-5901-6611
e-mail: hdilafruz_285@mail.ru

Abdukhalilova Shakhnoza Zafar kizi,
teacher of informatics,
School No. 29, Karshi city
ORCID ID: 0009-0008-1600-5283
e-mail: shaxnozaabduxalilova573@gmail.com

Abstract. The paper considers the problem of calculating the normal depth of steady uniform flow in an irrigation canal of rectangular cross-section on the basis of Manning's formula. The resulting nonlinear equation is solved by Newton's method, and the result is compared with the bisection method. The normal depth, flow velocity and Froude number are computed for various discharges and roughness coefficients, and the stage–discharge curve of the canal is constructed.

Keywords: irrigation canal, normal depth, Manning's formula, Newton's method, Froude number, stage–discharge curve.

Introduction. One of the most common problems in the design and operation of irrigation canals is determining the water depth in a canal for a given discharge. If the canal is long and its cross-section and slope are constant, **steady uniform flow** is established: the water depth does not change along the canal. This depth is called the **normal depth**; it is the key quantity for choosing the height of the canal banks, the

operating mode of gates and the water-measuring structures [1, 2].

The normal depth follows from a special case of the Saint-Venant equations, which describe unsteady flow in open channels. If the flow changes neither in time nor along the canal, only gravity and friction remain in the momentum equation, i.e. the friction slope equals the bed slope: $S_f = S_0$ [3]. The normal depth is also used as the initial state in numerical modelling of unsteady flow in rivers and canals. In particular, the initial hydraulic parameters in modelling the seasonal variability of the Zarafshan River flow and the flow around river spur dikes are determined in this way [5, 6].

Problem statement. For a canal of rectangular cross-section with width B, the flow area, wetted perimeter and hydraulic radius are

$$A = Bh, \qquad P = B + 2h, \qquad R = \frac{A}{P} = \frac{Bh}{B+2h}. \quad (1)$$

In steady uniform flow, the discharge is given by Manning's formula:

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}, \quad (2)$$

where Q is the discharge, m³/s; n is the roughness coefficient; S_0 is the bed slope. Substituting (1) into (2) gives a nonlinear equation in h:

$$f(h) = Q - \frac{\sqrt{S_0}}{n} Bh \left(\frac{Bh}{B+2h}\right)^{2/3} = 0. \quad (3)$$

This equation cannot be solved for h analytically, so a numerical method is required.

Solution method. Equation (3) is solved by Newton's method:

$$h_{k+1} = h_k - \frac{f(h_k)}{f'(h_k)}, \qquad k = 0,1,2, \ldots \quad (4)$$

The derivative has the analytical form

$$f'(h) = -\frac{\sqrt{S_0}}{n} B R^{2/3} \left(1 + \frac{2B}{3(B+2h)}\right). \quad (5)$$

For example, at $h = 1$ m we obtain $f'(1) = -7.92 \cdot \left(1 + \frac{20}{36}\right) = -12.32$. The iteration stops when $|h_{k+1} - h_k| < \varepsilon$; in the calculations $\varepsilon = 10^{-6}$ m. To check the result, the Froude number, which indicates the flow regime, and the critical depth are also computed:

$$Fr = \frac{u}{\sqrt{gh}}, \qquad u = \frac{Q}{Bh}, \qquad h_c = \sqrt[3]{\frac{q^2}{g}}, \qquad q = \frac{Q}{B}. \quad (6)$$

If $Fr < 1$ (i.e. $h > h_c$), the flow is subcritical; if $Fr > 1$, it is supercritical.

Worked example. Canal parameters: $B = 10$ m, $S_0 = 0.0005$, $n = 0.025$ (concrete lining), $Q = 20$ m³/s. The initial guess is $h_0 = 1$ m. At the first step $A = 10$ m², $R = 0.833$ m, $R^{2/3} = 0.886$, and Manning's formula gives a discharge of 7.92 m³/s. Hence $f(1) = 20 - 7.92 = 12.08$, $f'(1) \approx -12.32$ and

$$h_1 = 1 - \frac{12.08}{-12.32} = 1.980 \text{ m}. \quad (7)$$

The next steps give $h_2 = 1.8388$ m, $h_3 = 1.8367$ m, $h_4 = 1.8367$ m. Thus, **in four iterations** the normal depth $h_n = 1.837$ m was found. The flow velocity is then $u = 1.089$ m/s and the Froude number $Fr = 0.257$, i.e. the flow is subcritical.

Results. The same calculation was repeated for various discharges. For comparison, equation (3) was also solved by the bisection method on the interval from 0.01 m to 10 m. The results are given in Table 1.

Table 1. Calculation results for various discharges ($B = 10$ m, $S_0 = 0.0005$, $n = 0.025$)

Q, m³/s	h_n, m	u, m/s	Fr	h_c, m	Iterations (Newton)	Iterations (bisection)
5	0.746	0.670	0.248	0.294	4	24
10	1.163	0.860	0.255	0.467	4	24
15	1.516	0.989	0.257	0.612	4	24
20	1.837	1.089	0.257	0.742	4	24
25	2.136	1.170	0.256	0.860	5	24
30	2.421	1.239	0.254	0.972	5	24
35	2.694	1.299	0.253	1.077	5	24
40	2.958	1.352	0.251	1.177	5	24

The table shows that $h_n > h_c$ in all cases, i.e. the flow in the canal is subcritical. Newton's method converges in 4–5 iterations, while the bisection method requires 24

iterations. This is explained by the quadratic convergence of Newton's method: the number of correct digits roughly doubles at each step.

The influence of the roughness coefficient was also studied ($Q = 20$ m³/s). For a smooth concrete lining ($n = 0.014$) the normal depth is 1.251 m, for ordinary concrete ($n = 0.025$) it is 1.837 m, and for a weed-grown earth canal ($n = 0.035$) it is 2.308 m. Hence, vegetation growth or silting of a canal can raise the water level by up to one metre. This quantitatively justifies the need for regular canal maintenance.

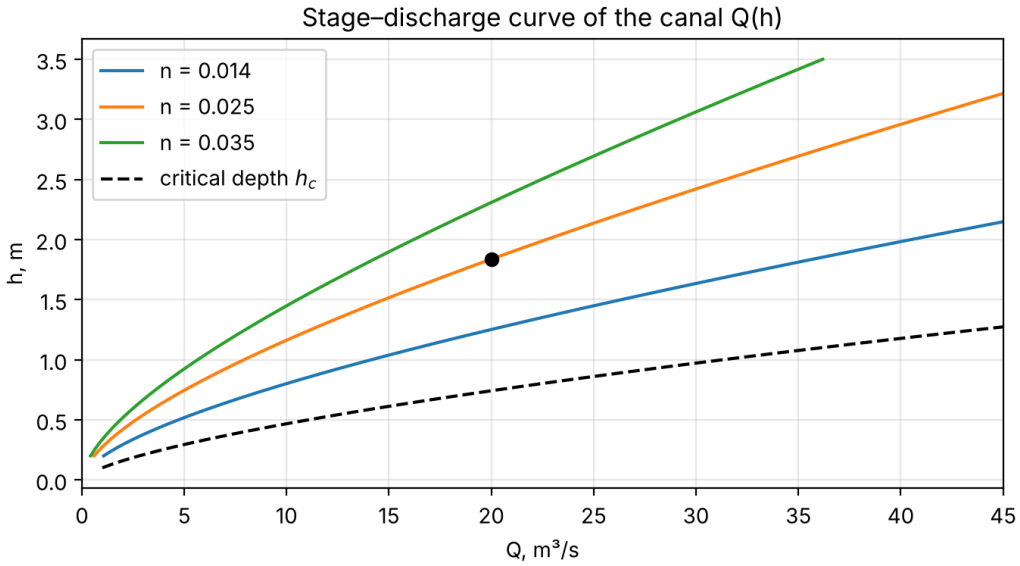


Figure 1. Stage–discharge curves $Q(h)$ of the canal for different roughness coefficients; dashed line – critical depth, dot – worked example

Figure 1 shows the stage–discharge curves of the canal. In hydraulic engineering practice, these curves allow the discharge passing through a canal to be determined quickly from a measured water level. The stage–discharge curve is also used as the downstream boundary condition in modelling unsteady flow.

Conclusions.

Determining the normal depth in a rectangular canal reduces to solving a nonlinear equation, which is efficiently solved by Newton's method.

Newton's method reaches an accuracy of 10^{-6} m in 4–5 iterations and is about 5

times faster than the bisection method.

The roughness coefficient strongly affects the normal depth: an increase of n from 0.014 to 0.035 increases the water depth by a factor of 1.8.

The proposed algorithm is simple and can be used by students as a practical exercise in the courses “Numerical Methods” and “Mathematical Modelling”.

References

1. Chow V.T. Open-Channel Hydraulics. – New York: McGraw-Hill, 1959. – 680 p.
2. Chanson H. The Hydraulics of Open Channel Flow: An Introduction. – 2nd ed. – Oxford: Butterworth-Heinemann, 2004. – 634 p.
3. Cunge J.A., Holly F.M., Verwey A. Practical Aspects of Computational River Hydraulics. – London: Pitman, 1980. – 420 p.
4. Burden R.L., Faires J.D. Numerical Analysis. – 9th ed. – Boston: Brooks/Cole, 2011. – 872 p.
5. Seytov A., Xamroyeva D. Modelling the seasonal and annual variability of the Zarafshan River flow using numerical methods // Applied Mathematics, Digital Transformation and Artificial Intelligence: Current Problems and Prospects: Abstracts of the International Scientific and Practical Conference, Karshi, June 5–6, 2026. – Karshi: Karshi State University, 2026. – P. 631–633. (in Uzbek)
6. Seytov A.J., Xamroyeva D.N., Kalxanov P.J. Numerical modelling of the flow around spur dikes of the Zarafshan River based on the two-dimensional Saint-Venant equations // Fan va jamiyat (Science and Society). – Nukus: Nukus State Pedagogical Institute named after Ajiniyaz, 2026. – No. 6/1. – P. 20–22. (in Uzbek)