GIS-BASED DIGITAL TERRAIN MODELING FOR MORPHOMETRIC ASSESSMENT OF SEASONAL STREAM BASINS IN MOUNTAINOUS REGIONS

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ABSTRACT

Seasonal streams in mountainous and foothill regions can provide an important source of runoff, particularly during snowmelt and periods of rainfall. However, their irregular flow regimes and complex terrain make the assessment of basin characteristics challenging. This study presents a mathematical approach to representing the topographic surface of a seasonal stream basin and deriving a digital terrain model from elevation reference points. The Achchiksay basin, located in the Kyzylcha area of Dehqonabad district, Kashkadarya region, Uzbekistan, was used as the case study. A calculation grid was formed from the available elevation data, and an interpolation-based procedure was applied to describe the spatial variation of the terrain. The mathematical formulation incorporates a series representation and integral transformations to obtain an expression for basin-surface modeling. A numerical experiment using the study site's experimental parameters was then carried out, resulting in a digital terrain model of the basin. The model provides a spatial representation of elevation and can support subsequent morphometric analysis and

preliminary assessment of seasonal runoff catchments. The approach may be useful in terrain studies where reliable representation of basin relief is needed to inform water-resource planning. Further validation against independent elevation data and field measurements would help quantify model accuracy.

Keywords: digital terrain model; seasonal stream basin; GIS; elevation data; morphometric assessment; interpolation; mathematical modeling; Achchiksay basin; mountainous regions.

INTRODUCTION

Seasonal streams are important components of the hydrological systems of mountainous and foothill landscapes. Their runoff is often concentrated in spring and during rainfall events, while flow may decrease substantially or cease in dry periods. The timing and spatial distribution of runoff are influenced by terrain, precipitation, snowmelt, and catchment characteristics. A reliable representation of basin relief is therefore a useful first step in analyzing the physical structure of a catchment and supporting water-resource planning. Digital terrain models (DTMs) provide a structured representation of elevation and allow terrain characteristics to be examined using a consistent spatial framework.

By water sources: rivers fed by snowmelt, rivers fed by glacial water, rivers fed by rainwater, rivers fed by groundwater, and rivers with mixed sources.

By water regime: rivers with continuous runoff, rivers with seasonal runoff, and rivers with temporary runoff.

By length: small rivers (up to 100 km), medium rivers (100–500 km), and large rivers (more than 500 km).

By catchment basin area: rivers with small basins, medium basins, and large basins.

Relief: mountain rivers, foothill rivers, and plain rivers.

Streams are small watercourses formed mainly in mountainous and foothill areas, most of which are seasonal. Flow in streams most often occurs in spring and during periods of heavy precipitation, while during the dry season the flow ceases.

By duration of runoff: permanent streams, seasonal streams, and ephemeral streams.

Seasonal streams are mainly widespread in the mountainous regions of Uzbekistan, and their water regime is closely related to climatic conditions.

By water sources: streams formed by rainwater, streams formed by snowmelt, streams fed by spring water, and streams with mixed sources.

By basin area: very small basin (up to 10 km²), small basin (10–100 km²), medium basin (100–500 km²), and large basin (over 500 km²).

By geomorphological location: mountain streams, foothill streams, and hill streams.

By hydrological characteristics: mudflow-forming streams, streams with low mudflow hazard, streams with high turbidity, and streams with low turbidity.

Digital Terrain Modeling and Morphometric Representation of a Seasonal Stream Basin

The construction of a digital terrain model is an important step in describing the relief and calculating the morphometric characteristics of a seasonal stream basin. In this study, the modeling task is treated through two related operations: representing the terrain surface at a defined level of spatial detail, and estimating surface values between known elevation points. These operations convert a continuous topographic surface into a discrete mathematical representation that can be used for further spatial analysis. The proposed workflow is examined for the Achchiksay basin in the Kyzylcha area of Dehqonabad district, Kashkadarya region.

Based on the above, modeling of the topographic surface includes two interrelated processes:

Discretization of the required surface by determining infinite or finite sets of points that fully or partially represent the topographic surface at the required level;

Graphical or mathematical (digital) representation of a continuous surface by determining a finite discrete set of points.

Thus, mathematical (digital) modeling of the relief of a seasonal stream basin means the representation or continuous-discrete transformation of continuous functions representing the parameters of the terrain into a discrete form based on specified conditions.

One of the effective methods for determining the surface area of topographic relief is the development of a differential digital surface model. Using this method, we determine the contour lines over the surface of the Achchiksay basin in the Kyzylcha area of Dehqonabad district, Kashkadarya region, which is the object of the study (Fig. 3.1). We then draw vertical lines along the reference points on the surface. As a result, the relief surface is covered by a rectangular grid. Suppose that the reference points on the surface are accurately and completely defined by a set of coordinates (x,y,z). Then the surface is modeled as a system of non-overlapping rectangles (Fig. 3.2).

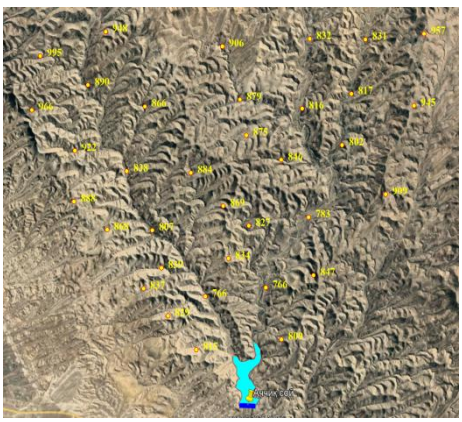


Figure 1. Elevation characteristics of the relief of the seasonal stream basin area.

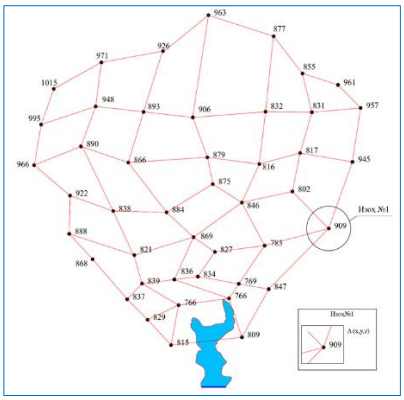


Figure 2. Experimental values of the morphometric parameters of the seasonal stream basin

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Description of Figure 2. The figure illustrates reference points connected to form the calculation grid for the terrain surface. The values shown beside the points represent elevation marks. These discrete observations provide the input for interpolation and mathematical representation of the basin topography.

Based on the problem conditions, let us seek the topographic surface of the seasonal stream basin area in the form of the function $\xi(x,y,z)$. For this purpose, we have the boundary-value problem for the transforms of this function in the following form:

$$\begin{cases} (\Delta^2+\epsilon^2)Z_{zz}-k^2\Theta^2 z=0 \\ (0<z<H),z=0 \\ Z=\phi(\mu,v),\ z=H \end{cases} \quad (1)$$

Here is the transform of the function: $\phi(\mu,v)$ - $f(x,y)$ the transform of the function $\Delta=\frac{\partial}{\partial z}+ik,\ k=(\mu,v).$

Here: $\phi(\mu,v)$ – transform of the function $f(x,y)$, $\Delta=\frac{\partial}{\partial z}+ik,\ k=(\mu,v).$

A set of interpolation functions corresponding to various parameters of the topographic relief of the seasonal stream basin has been found. In different interpolation functions, equation (1) has many different algorithms in the form of interpolation and splines, which differ significantly from each other.

The solution of system of equations (1) is sought in the form of the following series of the function Z at the reference points of the relief:

$$Z=\sum_{n=1}^{\infty} b_n(\mu,v)*Z_n(z) \quad (2)$$

Here: $b_n(\mu,v)$ - kernel function; Z_n - eigenfunction of the Sturm–Liouville problem.

From expressions (1) and (2), we obtain the following system of partial differential equations:

$$\begin{cases} \frac{\partial^2 Z_n}{\partial z^2}+\lambda_n^2\Theta^2 z_n=0 \\ Z_n(H)=Z_n(0)=0 \end{cases} (3)$$

Here: Θ - Weizsäcker–Brent function; ϖ_n^2- eigenvalue of the solution of the Sturm–Liouville problem.

n≠*m* имеем следующие условия:

$$\begin{cases} \int_H \Theta^2 Z_n Z_m dz=0 \\ \int_H \Theta^2 Z_n^2 dz=1 \end{cases} (4)$$

The coefficients b_n in series (2) are found in the form of Fourier series coefficients, that is:

$$b_n(\mu,v)= \int_H \Theta^2(z)Z(\mu,v,z)Z_n(z)dz (5)$$

Multiplying equation (1) by the eigenfunction $Z_n(z)$, and then integrating with respect to z, we obtain the expression for determining the coefficient b_n:

$$\begin{cases} b_n=Z_n(H)*\mathrm{h}*\phi(\mu,v)(\varpi_n^{-2}-k^2\varpi_n^{-4}b_{n1}), \\ \Delta^2 b_{n1}+(k^2\varpi_n^{-2}+\epsilon^2)b_{n1}=1 \end{cases} (6)$$

The solution of the simple differential equation (6) based on the boundary condition has the following form:

$$\begin{cases} b_{n1}=w_n^{-1}\int_z \sin(w_n\alpha)*exp[-i(\phi(z_0)-\phi(z))]dz \\ w_n^2=k^2\varpi_n^{-2}+\epsilon^2,\ \phi(z)=\int[\mu+v]\,d\xi \end{cases} (7)$$

Substituting expression (7) into (6) and taking (2) into account, we obtain:

$$\begin{cases} Z=\mathrm{h}\sum_{n=1}^{\infty}\varphi_n(z)\phi(\mu,v)\left(1-k^2\varpi_n^{-2}b_{n1}\right), \\ \varphi_n(z)=\varpi_n^{-2}z_n(z)*z_n(H) \end{cases} (8)$$

Applying the inverse Fourier transform method to system of equations (8), we obtain the following equation for the topographic surface of the seasonal stream basin area:

$$\Xi(x,y,z)=H\sum_{n=1}^{\infty}\varphi_n(z)[f(x,y)+J_n(x,y)], \quad (9)$$

Here

$$\begin{cases} J_n(x,y)=\varpi_n^{-2}\int_z I_n dz, \\ I_n=-\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k^2\phi(\mu,\nu)*\frac{\sin w_n\alpha}{w_n}\exp[i(\mu\Delta x+\nu\Delta y)]d\mu dv \\ \Delta x=x-\Delta x_0,\quad \Delta y=y-\Delta y_0, \\ \Delta x_0=\int_\xi \mu d\xi,\ \Delta y_0=\int \mu d\xi \end{cases} \quad (10)$$

Let us reduce the double integral in system of equations (10) to the following form:

$$I_n=-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Delta_2 f\left(\Delta x-x_1,\ \Delta y-y_1\right)\Lambda(R_1)\,dx_1 dy_1 \quad (11)$$

Let us write the function $\Lambda(R_1)$in equation (11) in the following form:

$$k=k_1\lambda_n, R_1^2=x_1^2+y_1^2 \quad (12)$$

Let us transform the parameters from equation (12) into the polar coordinate system: $\mu=k\cos\theta$ and $v=k\sin\theta$, then integrate it over the angle θ and, using the zero-order Bessel function, obtain a simpler expression for $G(R_1)$:

$$\begin{cases} \textbf{состояние 1:} \\ \text{если } \alpha>\varpi_n R_1 \text{ тогда} \\ \Lambda(R_1)=\frac{\varpi_n^2}{2\pi\sqrt{\alpha^2-\varpi_\tau^2 R_1^2}}\cos\left(f\sqrt{\alpha^2-\varpi_\tau^2 R_1^2}\right), \text{будет} \\ \textbf{состояние 2}: \\ \text{если } \alpha<\varpi_n R_1 \text{ тогда} \\ \Lambda(R_1)=0 \text{ будет} \end{cases} \quad (13)$$

To simplify the operations for solving equations (9) and (13), we seek the non-uniform distribution of the relief of the seasonal stream basin area in the following form:

$$\begin{cases} f(x,y)=\left(\frac{R^2}{L^2}+1\right)^{-m} \\ R^2=x^2+y^2 \end{cases} \quad (14)$$

Here: L is the parameter characterizing the horizontal scale of relief irregularities, and m is the parameter determining the degree of slope.

For a function of the form (14), the double integral in (10) can be replaced by a multiple integral.

To expand equation (14) according to (11), we first calculate the Laplace operator:

$$\begin{cases} \Delta_2 f(x,y)=\phi_m, \\ \phi_m=4m^2L^{-2}Y^{-m-1}-4m(m+1)L^{-2}Y^{-m-2}, \\ Y=R^2L^{-2}+1 \end{cases} \quad (15)$$

Then the argument of the function $f(\Delta x-x_1,\ \Delta y-y_1)$ in equation (11) takes the following form:

$$\begin{cases} Y=\left[(\Delta x-x_1)^2+(\Delta y-y_1)^2\right]L^{-2}+1=\Pi-P\cos\varphi, \\ \Pi=\left[(\Delta R)^2+R_1^2\right]L^{-2}+1, \quad P=2\Delta R R_1 L^{-2}, \\ \Delta R=\sqrt{(\Delta x)^2-(\Delta y)^2}, \\ 0<\varphi<2\pi \end{cases} \quad (16)$$

Then the following auxiliary integrals are calculated::

$$\begin{cases} k_m=\int_0^{2\pi} Y^{-m}d\varphi, \\ k_1=\dfrac{2\pi}{\sqrt{\Pi^2-P^2}} \\ k_{m+1}=-\dfrac{1}{m}\dfrac{\partial}{\partial\Pi}Z_m \end{cases} \quad (17)$$

Thus, we have the solution of equation (11) in the following form:

$$\Omega_m=(2\pi)^{-1}\left[4m^2*L^{-2}k_{m+1}-4m(m+1)L^{-2}k_{m+2}\right] \quad (18)$$

Introducing the following transformation $\eta=\sqrt{\alpha^2-\lambda_n^2R_1}$, we obtain the double integral for J_n in equations (12) and (18):

$$J_n=-\varpi_n^{-2}\int_0^z\int_0^\alpha \Omega_m\cos(l\eta)d\eta dz \quad (19)$$

By reducing expressions (18) and (19) to equation (9), we obtain the following solution:

$$\Xi(x,y,z)=H\sum_{n=1}^{\infty}\varpi_n^{-2}z_n(z)*z_n(H)\left[\left(\frac{R^2}{L^2}+1\right)^{-m}-\right. \qquad -$$

$$\left.\varpi_n^{-2}\int_0^z\int_0^{\alpha}\Omega_m cos(\epsilon\eta)d\eta dz\right],(20)$$

Consequently, expression (20) provides the mathematical basis used in this study to represent the surface of a seasonal stream basin and construct its digital terrain model.

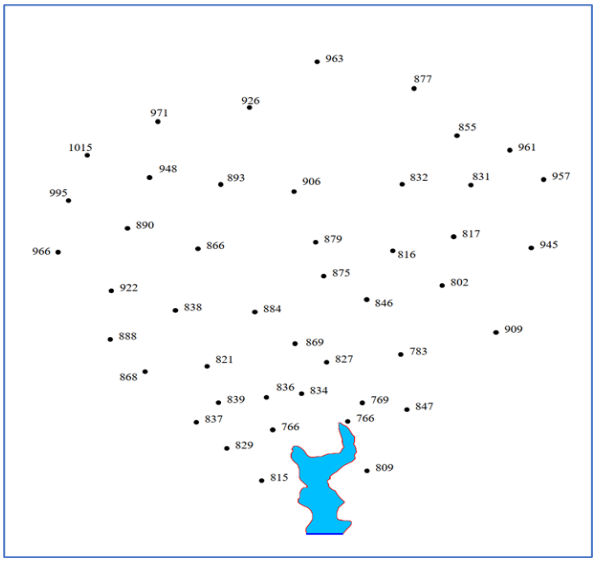


Figure 3. Elevation marks of the relief of the seasonal stream basin area

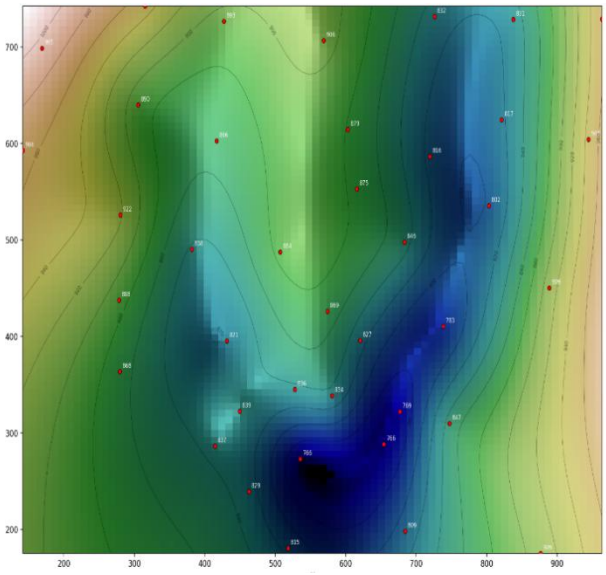


Figure 4. Digital terrain model of the seasonal stream basin

A numerical experiment was carried out using expression (20) and the experimental parameters available for the Achchiksay study area. The resulting surface representation was used to construct a digital terrain model of the basin (Figures 3–5). The experiment demonstrates the application of the proposed mathematical procedure to the selected study area; independent validation would be required to establish quantitative accuracy.

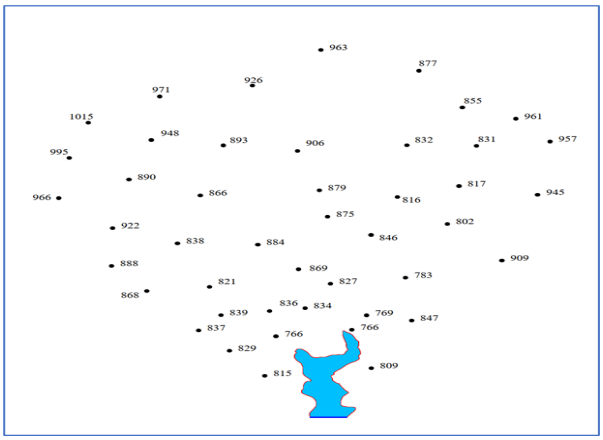


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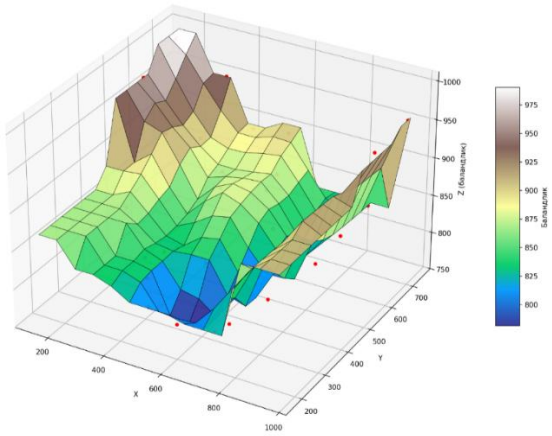


Figure 5. Digital terrain model (DEM) of the middle-mountain area formed in Achchiksay

CONCLUSION

This study considered a mathematical approach for constructing a digital terrain model of a seasonal stream basin using elevation reference points and experimental data from the Achchiksay basin in Uzbekistan. The reference points were organized into a calculation grid, and an interpolation-based formulation was used to represent the spatial variation of the topographic surface. A numerical experiment based on expression (20) produced a digital representation of the study area's relief. Such a model can provide a basis for subsequent morphometric assessment and support preliminary analysis of seasonal stream catchments for water-resource planning. The present results demonstrate the feasibility of applying the mathematical procedure to the selected basin, but do not by themselves establish a measured improvement in accuracy. Future work should compare the generated surface with independent elevation data, report quantitative error indicators, and examine how terrain-model resolution affects derived basin characteristics.

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