MODELLING OF THE WAVE GENERATED IN AN IRRIGATION CANAL BY GATE OPENING ON THE BASIS OF THE SAINT-VENANT EQUATIONS

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Abstract. The paper models the wave that propagates along an irrigation canal when the gate at the canal entrance is opened and the discharge increases sharply, using the one-dimensional Saint-Venant equations. The equations are solved by the finite volume method with the Rusanov scheme. The arrival time of the wave at canal cross-sections, the rise of the water level and the transition of the canal to a new steady state are determined. The numerical results are confirmed by the theory of characteristics.

Keywords: Saint-Venant equations, unsteady flow, irrigation canal, gate, finite volume method, CFL condition.

Introduction. In irrigation systems, water distribution is controlled by gates. When a gate is opened or closed, **unsteady flow** arises in the canal: a wave propagates along the canal, and the water level and discharge change in time. Knowing in advance when and how much water will reach the consumer is important for saving water

resources and preventing overtopping of canal banks [1]. The basic model describing such processes is the system of one-dimensional equations proposed by A. Saint-Venant in 1871 [2]. The Saint-Venant equations have been successfully applied to modelling the seasonal and annual variability of the Zarafshan River flow [5] and the flow around river spur dikes in a two-dimensional formulation [6]. In this paper, the same approach is applied to an irrigation canal in the one-dimensional case.

Mathematical model. For a canal of rectangular cross-section, the Saint-Venant equations in conservative form are

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0, \quad (1)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{h} + \frac{gh^2}{2}\right) = gh(S_0 - S_f), \quad (2)$$

where $h(x,t)$ is the water depth, m; $q = Q/B$ is the discharge per unit width, m²/s; S_0 is the bed slope; S_f is the friction slope given by Manning's formula:

$$S_f = \frac{n^2\, q|q|}{h^2 R^{4/3}}, \qquad R = \frac{Bh}{B+2h}. \quad (3)$$

Here equation (1) expresses conservation of mass and equation (2) conservation of momentum. The system is hyperbolic, and its characteristic speeds are

$$\lambda_{1,2} = u \mp \sqrt{gh}, \qquad u = \frac{q}{h}. \quad (4)$$

The quantity $\lambda_2 = u + \sqrt{gh}$ is the speed of the wave travelling downstream along the canal.

Problem statement. Canal parameters: length $L = 1000$ m, width $B = 10$ m, bed slope $S_0 = 0.0005$, roughness coefficient $n = 0.025$. Initially, steady uniform flow with discharge $Q_0 = 20$ m³/s exists in the canal; Manning's formula gives the normal depth $h_0 = 1.837$ m. At $t = 0$ the gate is opened and the discharge rises to $Q_1 = 30$ m³/s.

Boundary conditions: at the canal entrance $Q(0,t) = 30$ m³/s, and at the canal end the stage–discharge relation $Q(L,t) = Q_M(h(L,t))$ is prescribed. Initially $u_0 = 1.089$ m/s, $\sqrt{gh_0} = 4.245$ m/s and the Froude number $Fr = 0.257 < 1$, i.e. the flow is subcritical. For subcritical flow one condition must be prescribed at each boundary, which agrees with

the conditions above.

Numerical method. The canal is divided into cells of length Δx. The cell averages $U_i = (h_i, q_i)$ are updated by the scheme

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left(\hat{F}_{i+1/2} - \hat{F}_{i-1/2}\right) + \Delta t\, S_i^n, \quad (5)$$

where the numerical flux at cell interfaces is computed by the Rusanov formula [3, 4]:

$$\hat{F}_{i+1/2} = \frac{F_i + F_{i+1}}{2} - \frac{a_{i+1/2}}{2}(U_{i+1} - U_i), \qquad a_{i+1/2} = \max(|\lambda|_i, |\lambda|_{i+1}). \quad (6)$$

For stability of the explicit scheme, the time step is chosen from the Courant–Friedrichs–Lewy (CFL) condition:

$$\Delta t = \mathrm{CFL} \cdot \frac{\Delta x}{\max_i(|u_i| + \sqrt{g h_i})}, \qquad \mathrm{CFL} = 0.8. \quad (7)$$

For example, $\Delta x = 25$ m gives $\Delta t \approx 3.7$ s. The computations were carried out in Python using the NumPy library.

Results. First, the grid dependence of the numerical solution was checked (Table 1). When Δx is halved, the depth at $x = 500$ m changes by less than 0.01 m, i.e. the solution is practically grid-independent.

Table 1. Grid dependence of the results at $x = 500$ m

Δx, m	Steps	h, m (t = 300 s)	Q, m³/s (t = 300 s)	h, m (t = 600 s)	Q, m³/s (t = 600 s)	h, m (t = 1800 s)	Q, m³/s (t = 1800 s)
50	273	2.071	27.45	2.219	28.12	2.397	29.78
25	545	2.073	27.78	2.225	28.23	2.399	29.80
12.5	1089	2.071	28.00	2.229	28.29	2.400	29.82

Figure 1 shows the discharge hydrographs at three cross-sections of the canal. When the wave reaches a cross-section, the discharge rises sharply and then gradually approaches the new value of 30 m³/s.

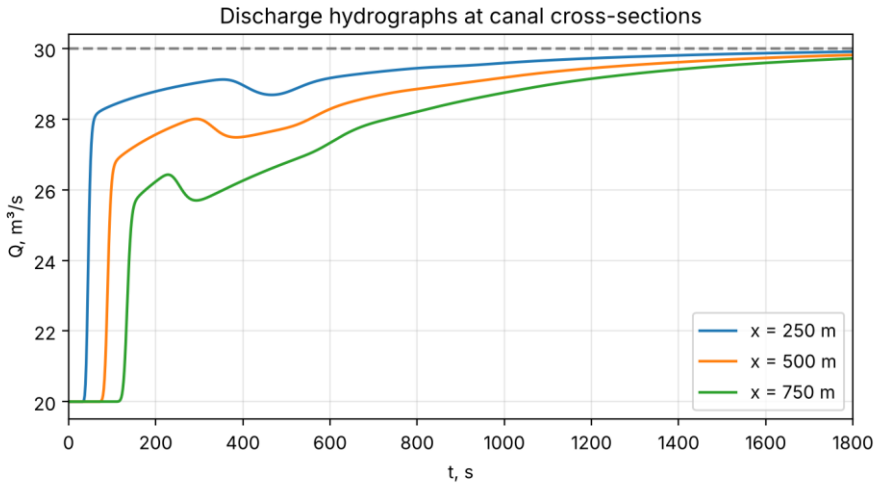


Figure 1. Discharge hydrographs at the cross-sections $x = 250$, 500 and 750 m ($\Delta x = 12.5$ m)

The arrival time of the wave at the cross-sections was compared with the theory of characteristics. The theoretical time is $t = x/\lambda_2$ with $\lambda_2 = 5.334$ m/s. In the numerical solution, the arrival time was taken as the moment when the discharge exceeds 25 m³/s (Table 2).

Table 2. Arrival time of the wave at canal cross-sections

Cross-section x, m	Theoretical time x/λ_2, s	Numerical result, s	Difference, %
250	46.9	47.2	0.6
500	93.7	94.3	0.6
750	140.6	144.6	2.8

The results differ by less than 3 %. The difference grows slightly with distance from the gate, because friction and numerical diffusion smear the wave front.

After about 30 minutes, the whole canal reaches a new steady state: water depth $h \approx 2.40$ m, discharge $Q \approx 29.8$ m³/s. This is close to the normal depth for $Q_1 = 30$ m³/s obtained from Manning's formula (2.421 m). Thus, when the discharge increases by a factor of 1.5, the water level in the canal rises by about 0.58 m, and the process lasts about half an hour.

Conclusions.

Unsteady flow generated in a canal by gate opening is modelled with sufficient accuracy using the one-dimensional Saint-Venant equations and the Rusanov scheme. The numerical solution is grid-independent: reducing Δx from 50 m to 12.5 m changes the depth by less than 0.01 m. The wave arrival time agrees with the theory of characteristics within 3 %; in a 1 km long canal the wave reaches the last cross-section in about 3 minutes. The complete transition of the canal to the new steady state takes about 30 minutes. This result can be used in scheduling gate operations and in notifying water users in advance.

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